DELAY-TIME MODEL BASED ON IMPERFECT INSPECTION OF AIRCRAFT STRUCTURE WITHIN FINITE TIME SPAN

Cai Jing, Zuo Hongfu, Zhu Lei

(College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, P. R. China)

Abstract: According to the failure characteristics of aircraft structure, a delay-time model is an effective method to optimize maintenance for aircraft structure. To imitate the practical situation as much as possible, imperfect inspections, thresholds and repeated intervals are concerned in delay-time models. Since the suggestion by the existing delay-time models that the inspections are implemented in an infinite time span lacks practical value, a delay-time model with imperfect inspection within a finite time span is proposed. In the model, the nonhomogenous Poisson process is adopted to obtain the renewal probabilities between two different successive inspections on defects or failures. An algorithm is applied based on the Nelder-Mead downhill simplex method to solve the model. Finally, a numerical example proves the validity and effectiveness of the model.

Key words: aircraft structure; delay-time model; imperfect inspection; optimal maintenance; finite time

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INTRODUCTION

Maintenance review board (MRB) report is one of the vital documents for the evaluation on the continuing airworthiness of civil aircrafts. A civil aircraft includes two main parts: the systems and the structure. The systems are usually imported together with a detailed maintenance analysis, while the structure is designed and manufactured by domestic researchers and the structure analysis thus is very important. Structural significant items (SSIs) in the structure are most important and need to be analyzed by MSG-3. Since most of the SSIs have the failure characteristics of delay-time, a delay-time model can be adopted to implement the analysis.

Most of the existing inspection optimization models[1-4] assumed the inspection with the same inspection intervals. The assumption facilitated the modeling but failed to fit in the practical situation. Thresholds and repeated intervals should be considered together.

The structure consists of many components with multiple failure modes. Each of the existing delay-time models was designed for a single failure mode. And the existing delay-time models assumed that the number of previous inspections was very large and the inspections were perfect, which did not match the reality that the life time of the structure is limited and the inspections are imperfect over a finite time span.

Therefore it is necessary to develop a new delay-time model with imperfect inspection within a a finite time span for aircraft structure.

1 NONHOMOGENOUS POISSON PROCESS

Structure inspection has been widely developed as a fault counting process, which has been proved as a nonhomogenous Poisson process (NHPP)[5-7]. The nonhomogeneous process $\{N(t),$

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E-mail: caijing@nuaa.edu.cn

$t \geqslant 0\}$ is with an intensity function $r(t)$, and satisfies:

(1) $P(N_0=0)=1$,

(2) $\forall$ $t \geqslant 0$ and $h>0$, if $h \to 0$, then

$P(N_{t,t+h} \geqslant 2)=o(h)$,

(3) Independent increment in the process,

(4) $P\{N(t+h)-N(t)=1\}=r(t)h+o(h)$.

If $N(t)$ follows a Poisson distribution with mean value function $m(t)$

$$P\{N(t)=k\}=\frac{[m(t)]^k}{k!}\mathrm{e}^{-m(t)}$$
$$k=0,1,2,\cdots \tag{1}$$

The mean value function $m(t)$, which is the expected number of faults emerging until a certain time point t, can be expressed in terms of the fault rate of the program

$$m(t)=\int_0^t r(t)\mathrm{d}t \tag{2}$$

2 DELAY-TIME MODEL FOR AIRCRAFT STRUCTURE

2.1 Assumption

A large number of failure modes emerge in aircraft structure. And the correction of one defect or failure has nominal impact on the steady state of the overall failure characteristics. To describe the characteristic of aircraft structure, the following assumptions are given:

(1) Because of fatigue damage and environmental degradation, the failure rate of aircraft structure exhibits aging effect (increasing failure rate).

(2) According to the use of thresholds and repeated intervals in aircraft maintenance program, the first inspection (threshold) takes place at kT time (k is a positive integer), then a repeated inspection takes place at each T time unit, and the time of inspection is negligible.

(3) According to NHPP, defects arise with the rate of defect occurrence $r(t)$, per unit time.

(4) Since the flight cycles (FC) have effect on the aircraft structure, defects and failures only arise while the aircraft is operating.

(5) The minimal repair will be carried out when the defects or failures are found. After a minimal repair, the structure is as old as before.

Because many factors (such as the level of inspections, lighting conditions, surface conditions, material thickness and edge effect, access to view, human error) have an important influence on inspections, it is assumed that inspections are imperfect. For example, if a defect is present at an inspection, there is a probability p that the defect can be identified, which implies that there is a probability $1-p$ that the defect can be unnoticed.

2.2 Analysis of NHPP

It has been proved that the failure process over each inspection interval is an NHPP (Fig. 1), and not identical over all the inspection intervals of the system, because all inspections are within a finite time span T_a.

Because the total life of the aircraft structure is T_a, the maximum number of the inspections within $[0,T_a]$ is

$$N=\mathrm{int}(T_a/T) \tag{3}$$

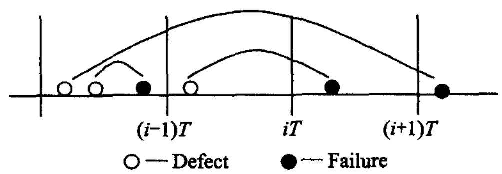

Fig. 1 Failure process in three imperfect inspections at points $(i-1)T$, iT, and $(i+1)T$

Fig. 1 shows that the expected number of the defects occurs within $[t,t+\delta t]$, $(i-1)T \leqslant t < iT$, is $r(t)\cdot\delta t$. If the inspection is perfect, the expected value of the failures caused by these defects is $r(t)\cdot\delta t\cdot F(iT-t)$. Therefore the expected value of the failures within $[(i-1)T, iT]$ can be obtained by integrating t into $[(i-1)T, iT]$.

$$E[N_f((i-1)T,iT)]=\int_{(i-1)T}^{iT} r(t)\cdot F(iT-t)\mathrm{d}t \tag{4}$$

But not all the defects can be found because of the limitation of imperfect inspections. The following appropriate analysis is given to develop another way to obtain the expected value of the failures.

(1) It is assumed that the defects emerge within $[0, kT]$.

If the defects is found at kT, then

$$E_{i,j}^{s} = \int_0^{kT} r(t) \cdot [1 - F(kT - t)] \mathrm{d}t \quad (5)$$

where $E_{i,j}^{s} (1 \leqslant i \leqslant j \leqslant N)$ is the expected number of the defects found at the jth inspection, when the defects emerge between the $(i-1)$th and the ith inspection.

If the defects are found at $(k+j-1)T, j=2, \cdots, N-k+1$, then

$$E_{1,j}^{s} = p^{j-1} \cdot \int_0^{kT} r(t) \cdot [1 - F((k+j-1)T - t)] \mathrm{d}t \quad (6)$$

If the failures happen within $[0, kT]$, then

$$E_{1,1}^{f} = \int_0^{kT} r(t) \cdot F(kT - t) \mathrm{d}t \quad (7)$$

If the failure is found within $[(k+j-2)T, (k+j-1)T], j=2, \cdots, N-k+1$, therefore

$$E_{1,j}^{f} = p^{j-1} \cdot \int_0^{kT} r(t) \cdot [F((k+j-1)T - t) - F((k+j-2)T - t)] \mathrm{d}t \quad (8)$$

where $E_{i,j}^{f} (1 \leqslant i \leqslant j \leqslant N)$ is the expected number of the failures found within the $(j-1)$th and the jth inspection, when the defects emerge between the $(i-1)$th and the ith inspection.

(2) It is assumed that the defects emerge within $[(k+i-2)T, (k+i-1)T]$, $i=2, \cdots, N-k+1$.

If the defects are found at $(k+i-1)T$, then

$$E_{i,i}^{s} = \int_{(k+i-2)T}^{(k+i-1)T} r(t) \cdot [1 - F((k+i-1)T - t)] \mathrm{d}t \quad (9)$$

If the defects are found at $(k+j-1)T$, $2 \leqslant i < j \leqslant N-k+1$, then

$$E_{i,j}^{s} = p^{j-i} \cdot \int_{(k+i-2)T}^{(k+i-1)T} r(t) \cdot [1 - F((k+j-1)T - t)] \mathrm{d}t \quad (10)$$

If the failures happen within $[(k+i-2)T, (k+i-1)T]$, then

$$E_{i,i}^{f} = \int_{(k+i-2)T}^{(k+i-1)T} r(t) \cdot F((k+i-1)T - t) \mathrm{d}t \quad (11)$$

If the failure is found within $[(k+j-2)T, (k+j-1)T]$, $2 \leqslant i < j \leqslant N-k+1$, then

$$E_{i,j}^{f} = p^{j-i} \cdot \int_{(k+i-2)T}^{(k+i-1)T} r(t) \cdot [F((k+j-1)T - t) - F((k+j-2)T - t)] \mathrm{d}t \quad (12)$$

The probabilities of all the repairs for the defects are obtained as the following matrix

$$\begin{bmatrix} E_{1,1}^{s} & E_{1,2}^{s} & \cdots & E_{1,N}^{s} \\ 0 & E_{2,2}^{s} & \cdots & E_{2,N}^{s} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{N,N}^{s} \end{bmatrix}$$

The probabilities of all minimal repairs for the failures are obtained as the following matrix

$$\begin{bmatrix} E_{1,1}^{f} & E_{1,2}^{f} & \cdots & E_{1,N}^{f} \\ 0 & E_{2,2}^{f} & \cdots & E_{2,N}^{f} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{N,N}^{f} \end{bmatrix}$$

Therefore the total expected repair cost for the defects is given as

$$E_1(C) = c_d \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} E_{i,j}^{s} \quad (13)$$

where c_d is the repair cost of a defect.

The total expected repair cost for the failures is given as

$$E_2(C) = c_f \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} E_{i,j}^{f} \quad (14)$$

where c_f is the repair cost of a failure.

The total expected cost within a finite time span T_a is given as

$$E(C) = N \cdot c_i + c_p + E_1(C) + E_2(C) \quad (15)$$

where c_i is the inspection cost and c_p the preventive maintenance cost.

The expected cost rate is given as

$$E_C(k, T) = \frac{E(C)}{E(T)} = \frac{N \cdot c_i + c_p + E_1(C) + E_2(C)}{N \cdot T} \quad (16)$$

2.3 Solution

To solve the optimal inspection interval T, the coefficient k of threshold inspection is involved. A step-by-step algorithm proposed by Li and Pham[4] based on the Nelder-Mead downhill simplex method is summarized in Fig. 2, where $\bar{f}$ is an average value.

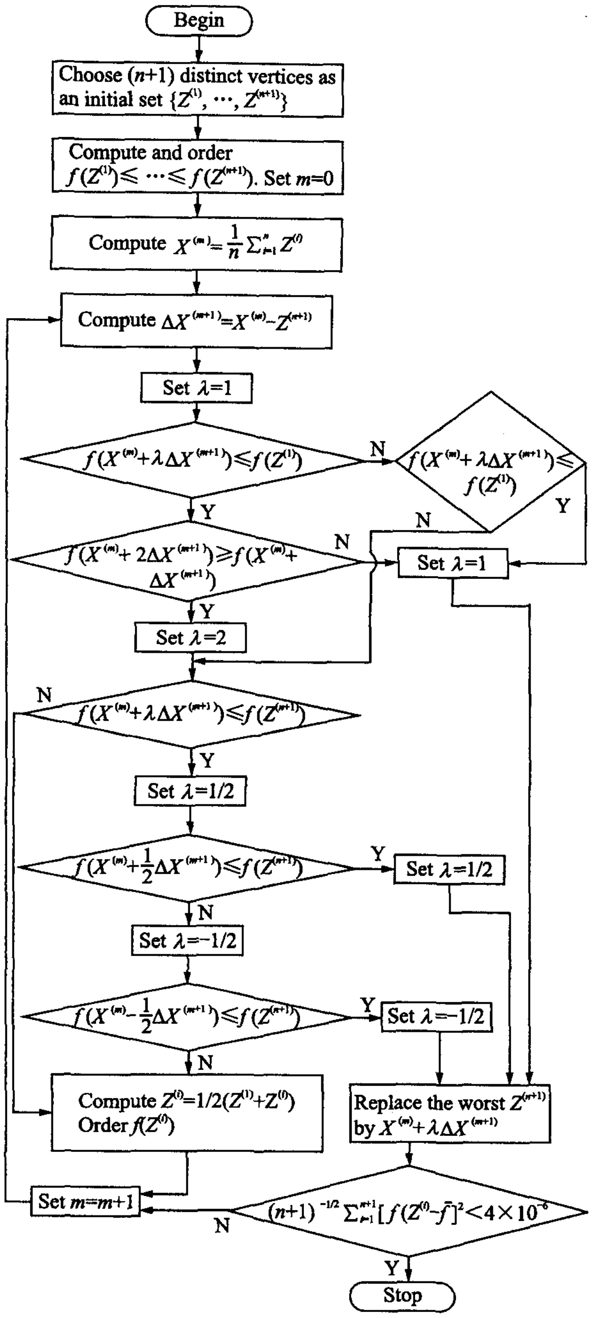

Fig. 2 Algorithm flow

3 NUMERICAL EXAMPLE

The validation takes a piece of fuselage skin as an example[8-9]. Because the statistical data of cracks from airlines have not been subdivided into three categories: fatigue crack, stress corrosion crack and corrosion fatigue crack, the causes of cracks are not been considered. Through parameter estimation and hypothesis testing, we can get $r(t)=\frac{1}{n}\left(\frac{t}{n}\right)^{(m-1)}$, $F(t)=1-\exp\left(-\left(\frac{t}{n}\right)^{m}\right)$, $m=1.21$, $n=14\ 642$, $c_i=200$ RMB, $c_d=1\ 000$ RMB, $c_f=4\ 000$ RMB, $c_p=2\ 000$ RMB, $p=0.9$, $T_a=20\ 000$ FC.

The values of k and T are now determined, therefore the average total cost per unit time $E_C(k,T)$ is minimized.

Since there are two decision variables k and T, $(n+1)=3$ initial distinct vertices are needed, which are $Z^{(1)}=(2,4\ 000)$, $Z^{(2)}=(4,2\ 000)$, $Z^{(3)}=(9,1\ 000)$. Set $m=0$. Based on Fig. 2, the optimal values (Table 1) are obtained.

Table 1 illustrates the process of the Nelder-Mead algorithm. It shows that a set of the optimal values is $k^*=5.24$, $T^*=2\ 056$ and the corresponding cost value is $E_C^*(5.24,205\ 6)=0.256\ 7$. If the proposed delay-time model is not used, namely, the total maintenance cost rate is calculated without considering the imperfect inspection and finite time, as described in general aircraft structure maintenance program, the threshold 6 000 FC is twice as much as the repeated interval 3 000 FC and the total maintenance cost rate

Table 1 Optimal values k, T

m	$Z^{(1)}$	$Z^{(2)}$	$Z^{(3)}$	Compute result
0	$E_C(2,4\ 000)=0.336\ 2$	$E_C(4,2\ 000)=0.297\ 7$	$E_C(9,1\ 000)=0.309\ 1$	$E_C(4.25,2\ 750)=0.301\ 3$
1	$E_C(4,2\ 000)=0.297\ 7$	$E_C(4.25,2\ 750)=0.301\ 3$	$E_C(9,1\ 000)=0.309\ 1$	$E_C(6.56,1\ 693)=0.289\ 9$
2	$E_C(6.56,1\ 693)=0.289\ 9$	$E_C(4,2\ 000)=0.297\ 7$	$E_C(4.25,2\ 750)=0.301\ 3$	$E_C(4.78,2\ 301)=0.280\ 7$
3	$E_C(4.78,2\ 301)=0.280\ 7$	$E_C(6.56,1\ 693)=0.289\ 9$	$E_C(4,2\ 000)=0.297\ 7$	$E_C(4.86,1\ 998)=0.276\ 2$
4	$E_C(4.86,1\ 998)=0.276\ 2$	$E_C(4.78,2\ 301)=0.280\ 7$	$E_C(6.56,1\ 693)=0.289\ 9$	$E_C(5.70,1\ 922)=0.268\ 3$
5	$E_C(5.70,1\ 922)=0.266\ 3$	$E_C(4.86,1\ 998)=0.276\ 2$	$E_C(4.78,2\ 301)=0.280\ 7$	$E_C(5.03,2\ 130)=0.268\ 4$
6	$E_C(5.70,1\ 922)=0.266\ 3$	$E_C(5.03,2\ 130)=0.268\ 4$	$E_C(4.86,1\ 998)=0.276\ 2$	$E_C(5.12,2\ 027)=0.269\ 7$
7	$E_C(5.70,1\ 922)=0.266\ 3$	$E_C(5.03,2\ 130)=0.268\ 4$	$E_C(5.12,2\ 027)=0.269\ 7$	Stop

is $E_C(2, 3\,000) = 0.3142$ which is more expensive than $E_C^*(5.24, 205\,6) = 0.2567$. Thus, it is proved that the proposed delay-time model is effective.

Table 1 shows that the bigger the optimized variable inspection interval T or threshold coefficient k is, the higher the probability of the defects or failure will be, which results in a higher maintenance cost. When the optimized variable inspection interval T or threshold coefficient k is smaller, which means there are more inspections, the frequent inspections will reduce the probability of failure, while incurring additional cost. So the choice of T or k must be appropriate to minimize the maintenance cost rate.

4 CONCLUSIONS

(1) The delay-time model within a finite time span is first presented and applied.

(2) The presented model is consistent with the practical situation of the aircraft structural maintenance.

(3) The example suggests that the threshold interval should be longer than the repeated inspection interval for the degraded system with increasing failure rate.

References:

[1] Lu H J. The use of degradation measures in assessing reliability[D]. Ames, USA: Iowa State University, 1992.

[2] Chao Chienmin. A study on modeling functional inspection at two stages[J]. Systems Engineering and Electronics, 2000,22(12):49-51. (in Chinese)

[3] Sachon M, Cornell E P. Delays and safety in airline maintenance[J]. Reliability Engineering and System Safety, 2000,67(3):301-309.

[4] Li W, Pham H. An inspection-maintenance model for systems with multiple competing processes[J]. IEEE Trans Reliab, 2005,54(2):318-327.

[5] Iskandar B, Murthy D, Jack N. A new repair-replace strategy for items sold with a two-dimensional warranty[J]. Comput Oper Res, 2005,32:669-682.

[6] Baik J, Murthy D N P, Jack N. Two-dimensional failure modeling with minimal repair[J]. Nav Res Log, 2004,51(3):345-62.

[7] Pham H, Zhang X. An NHPP software reliability model and its comparison[J]. International Journal of Reliability, Quality and Safety Engineering, 1997,4(3):269-282.

[8] Wang Hua, Zhou Xianbin. Structure parameters optimization on composite flexible pad for aircraft skin stretch forming with reconfigurable tool[J]. Acta Aeronautica et Astronautica Sinica, 2007, 28(6): 1482-1486. (in Chinese)

[9] Tan Xiaoming, Chen Yueliang, Duan Chengmei. Analysis of growth characterization of 3-D cracks in corroded lap joints of aircraft structure[J]. Acta Aeronautica et Astronautica Sinica, 2005,26(1):66-69. (in Chinese)

基于有限时间和不完备检测的飞机结构延迟模型

蔡 景 左洪福 朱 磊

(南京航空航天大学民航学院,南京,210016,中国)

摘要:延迟模型是针对飞机结构的故障特点,通过优化飞机结构的维修活动,以达到维修成本最优化的一种有效方法。基于现有的延迟模型都是建立在无限时间跨度假设上的不足,提出和建立了有限时间跨度上的飞机结构不完备检测延迟模型。模型中考虑了飞机结构维修中存在不完备检测、检测门槛值和重复检测间隔等情况,使得延迟模型更加符合飞机结构维修的实际情况。针对模型的复杂性,采用非齐次泊松过程方法研究两次检测之间的更新概率,并采用Nelder-Mead单纯形法对模型进行了优化求解,最后,通过数值算例,验证了模型的合理性和有效性。

关键词:飞机结构;延迟模型;不完备检测;维修优化;有限时间

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